GANIT *J. Bangladesh Math. Soc.* (ISSN 1606-3694) **35** (2015) 87-94

AN EVALUATION OF AN IMPROPER INTEGRAL ARISES FROM AN ANALYTIC SOLUTION OF A MODEL BOLTZMANN EQUATION FOR PHOTON TRANSPORT

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Received 22.03.2015 Accepted 18.08.2015

ABSTRACT

In this article we adopted the Mathematical model of solution of an improper integral which is created from the solution of the Boltzmann Transport equation (BTE) for photons. For the dose calculation of radiotherapy for cancer treatment, we need to solve the Boltzmann Transport equation. This improper integral is the important part of the BTE. Also the calculating time of the dose calculation is mostly dependent on the calculating time of this improper integral. For reducing the calculating time we need the minimum integrating area which is explained in this paper.

Keywords: Boltzmann Transport equation, Support, Radiotherapy, Minimum rectangle, Scattering cross section, Compton scattering

1. Introduction

It is most important to calculate the expected dose distribution of high energy photon radiotherapy for cancer treatment, before starting the treatment of the patient. If the dose of radiotherapy in the tumour tissue is not very low then we can expect curative effect. But if the dose is so high then the many healthy tissue surrounding the tumour will be destroyed or they will not be able to protect or avoid the undesirable side effect from the high dose. Therefore, one of the main parts for a treatment plan is the perfect dose calculation before beginning the treatment for effective the real treatment.

The Monte Carlo (MC) algorithm [2] has been used the exact dose calculation for photon and electron radiation by well known physical principles of interaction of radiation with human tissue by the transport of energy into the patient's body. If we work carefully then its leads to the exact results of the dose distribution in arbitrary geometries and nowadays highly developed MC codes for dose calculations are available. But in this process the computational time is very high. Therefore this process is becoming unattractive day by day in clinical use.

The alternative approach to circumvent the drawback of the MC codes called kernel models [1] offer a reliable and fast alternative for most types of radiation treatment. The pencil beam models are probably most in use and these models are based on the Fermi-Eyges theory of radiative transfer [10] and [5]. Pure electron radiation has been introduced by Hogstramet Mills and Almond [9] and later generalized to photon radiation by Gustafsson, Lind and Brahme [7] & Ulmer and Harder [14] too. Although the result was good but this models fail in complicated setting like air cavities or other inhomogeneities.

The third access for dose calculation is the deterministic Boltzmann equation of radiative transfer based on the physical interactions of radiation in tissue which is attracted in the last few years.

A mathematical model can be developed that allows in principle an exact dose calculation like as MC models. The resent studies for pure electron radiation were mostly done by Börgers and E.W. Larsen [3]. Electron and combined photon and electrion radiation were studied by Tervo et. al. [2]. Tervo and Kolmonen [13] in the context of inverse therapy planning and Zhengming et. al. [15] restricted their model to one dimensional slab geometry.

In the mathematical model [11] for the solution of the Boltzmann transport equation of photons we get an improper integral. In this paper we represent the mathematical model for the solution of this improper integral.

2. The model Boltzmann equation for photon transport [8]

The photons move with high velocities so all the process can be regarded as time independent and the all calculations are done relativistic using the relativistic formulae for energy and fully relativistic scattering cross section. For convenience all energies are scaled by the rest energy of the electron $mc^2 = 0.511\,\text{MeV}$, *m* being the rest mass of the election *c* is the velocity of light.

Let $\psi_\gamma(r,\Omega_\gamma,\varepsilon_\gamma)\ \cos\theta dAd\Omega_\gamma\ d\varepsilon_\gamma dt$ be the number of photons that move in time *dt* through area dA into the element of solid angle $d\Omega_\gamma$ around Ω_γ with an energy in the interval $(\varepsilon_\gamma,\varepsilon_\gamma + d\varepsilon_\gamma)$. θ is the angle between direction Ω_γ and outer normal of *dA*. $\Omega_\gamma = (\sin\varphi_\gamma \cos\vartheta_\gamma, \sin\varphi_\gamma \sin\vartheta_\gamma, \cos\varphi_\gamma)^T$ where φ_γ is the zenith angle and ϑ_γ is the polar angle in Cartesian coordinate system.

The Boltzmann transport equation for photons is

$$\Omega_\gamma.\nabla\psi_\gamma(r,\Omega_\gamma,\varepsilon_\gamma) = \rho_e(r)\int_0^\infty \int_{S^2} \tilde{\sigma}_{c,\gamma}(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma)\psi_\gamma(r,\Omega'_\gamma,\varepsilon'_\gamma)d\Omega'_\gamma d\varepsilon'_\gamma$$
$$-\rho_e(r)\sigma_{c,\gamma}^{tot}(\varepsilon_\gamma)\psi_\gamma(r,\Omega_\gamma,\varepsilon_\gamma) \tag{1}$$

where ρ_e is the electrons density of the medium and $\tilde{\sigma}_{c,\gamma}$ is the scattering cross section of the photons, differential in angle and energy for comptons scattering of photons and $\sigma_{c,\gamma}^{tot}(\varepsilon_\gamma)$ is the total compoton scattering cross section of photons.

2.1 Solution of the improper intergral in the interval $[0,\infty)$

When $i \ge 1$ then $[11]$ the solution of (1) is

$$\psi_\gamma^{(i)}(r(\lambda),\Omega_\gamma,\varepsilon_\gamma) = \int_0^\lambda g_{\varepsilon_\gamma,\Omega_\gamma}^{(i-1)}(t)e^{-\left(\int_t^\lambda \rho_e(r(s))ds\right)\sigma_{c,\gamma}^{tot}(\varepsilon_\gamma)}dt, \tag{2}$$

where we use the notation

$$g^{(i-1)}_{\varepsilon_\gamma,\Omega_\gamma}(\lambda)=\rho_e(r(\lambda))\int_o^\infty \int_{s^2}\tilde{\sigma}_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)\psi^{(i-1)}_\gamma\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)d\Omega'_\gamma d\varepsilon'_\gamma. \tag{3}$$

In the formula of $\psi^{(i)}_\gamma$, for $i\geq 1$, we see that $g^{(i-1)}_{\varepsilon_\gamma,\Omega_\gamma}$ contains the integral

$$\int_o^\infty \int_{s^2}\tilde{\sigma}_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)\psi^{(i-1)}_\gamma\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)d\Omega'_\gamma d\varepsilon'_\gamma. \tag{4}$$

We get from [4]

$$\tilde{\sigma}_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=\sigma_{c,\gamma}\left(\varepsilon'_\gamma,\Omega'_\gamma.\Omega_\gamma\right)\delta_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right) \tag{5}$$

With

$$\sigma_{c,\gamma}\left(\varepsilon'_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=\frac{r_e^2}{2}\left[\frac{1}{1+\varepsilon'_\gamma(1-\cos\theta)}\right]^3+\left[1+\cos^2\theta\,\frac{\varepsilon'^2_\gamma(1-\cos\theta)^2}{1+\varepsilon'_\gamma(1-\cos\theta)}\right] \tag{6}$$

and

$$\delta_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=\delta\left[\varepsilon_\gamma-\frac{\varepsilon'_\gamma}{1+\varepsilon'_\gamma(1-\cos\theta)}\right]. \tag{7}$$

In the expression above, $\cos\theta=\Omega'_\gamma.\Omega_\gamma$.

We know from [6] that $\int_o^\infty G\left(\varepsilon'_\gamma\right)\delta\left(h\left(\varepsilon'_\gamma,\varepsilon_\gamma\right)\right)d\varepsilon'_\gamma=\sum\dfrac{G\left(E'_\gamma\right)}{\left|\dfrac{\partial h}{\partial\varepsilon_\gamma}\left(E'_\gamma,\varepsilon_\gamma\right)\right|}$, (8)

Where the sum is taken over all those $E'_\gamma\in(0,\infty)$ such that $h\left(E'_\gamma,\varepsilon_\gamma\right)=0$. In case there exist no E'_γ in $(0,\infty)$ such that $h\left(E'_\gamma,\varepsilon_\gamma\right)=0$ then.

$$\int_o^\infty G\left(\varepsilon'\right)\delta\left(h\left(\varepsilon'_\gamma,\varepsilon_\gamma\right)\right)d\varepsilon'_\gamma=0.$$

Here

$$h\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=\varepsilon_\gamma-\frac{\varepsilon'_\gamma}{1+\varepsilon'_\gamma(1-\cos\theta)}.$$

Now we will find the value of incoming energy ε'_γ for given ε_γ and $\Omega'_\gamma.\ \Omega_\gamma$ such that for this value the function h will be zero. Let $\varepsilon'_\gamma=E'_\gamma$ be that value. Therefore

$$h\left(E'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=0 \tag{9}$$

Also we need

$$\frac{\partial h}{\partial\varepsilon'_\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=\frac{-1}{\left[1+\varepsilon'_\gamma(1-\cos\theta)\right]^2}. \tag{10}$$

Then we proceed to the calculation of those $E'_\gamma\in(0,\infty)$ such that $h\left(E'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)=0$

(for given $\varepsilon_\gamma \in (0,\infty)$ and given $\cos\theta = \Omega'_\gamma . \Omega_\gamma \in [-1,1]$).

Therefore, we fix $\varepsilon_\gamma \in (0,\infty)$ and $C = \cos\theta \in [-1,1]$ and we define $f(x) := h(x,\varepsilon_\gamma,C) = \varepsilon_\gamma - \frac{x}{1+x(1-C)}$ for $x \in (0,\infty)$. We can get the value of E'_γ by solving the equation $f(x) = 0$.

1. Case C=1.

$$f(x) = \varepsilon_\gamma - x = 0 \Leftrightarrow x = \varepsilon_\gamma. \tag{11}$$

Let us call $\tilde{E}'_\gamma = \varepsilon_\gamma$

2. Case $C \neq 1$.

Then we get the following:

$$f(x) = \varepsilon_\gamma - \frac{x}{1+x(1-C)} \text{ for } x \in (0,\infty), \text{and } f(0) = \varepsilon_\gamma > 0 \tag{12}$$

$$\lim_{x\to\infty} f(x) = \varepsilon_\gamma - \frac{1}{1-C} \tag{13}$$

$$\text{and } f'(x) = \frac{-1}{[1+x(1-C)]^2} < 0 \tag{14}$$

(a) When $\varepsilon_\gamma < \frac{1}{1-c}$ then

$\exists! \bar{x} \in (0,\infty)$ such that $f(\bar{x}) = 0$ and $\bar{x} = \frac{\varepsilon_\gamma}{1+\varepsilon_\gamma(c-1)}$ (see Figure 1). Let us call $E'_\gamma = \frac{\varepsilon_\gamma}{1+\varepsilon_\gamma(c-1)}$.

(b) When $\varepsilon_\gamma \geq \frac{1}{1-c}$ then

there is no $\bar{x} \in (0,\infty)$ such that $f(\bar{x}) = 0$ (see Figure 2).

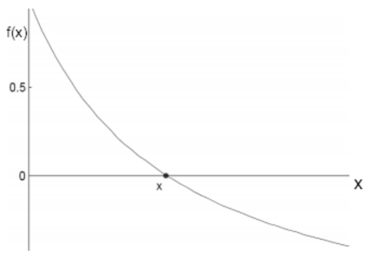

Figure 1

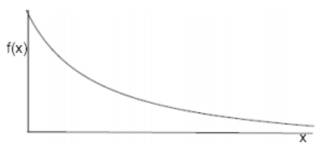

Figure 2

Also we need:

1. When $C=\cos\theta=\Omega'_\gamma.\,\Omega_\gamma=1$ then

$$\frac{\partial h}{\partial\varepsilon'_\gamma}\left(\tilde{E}'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\,\Omega_\gamma\right)=\frac{\partial h}{\partial\varepsilon'_\gamma}\left(\tilde{E}'_\gamma,\varepsilon_\gamma,1\right)=-1. \tag{15}$$

2. When $c\in[-1,1]$ and $\varepsilon_\gamma<\dfrac{1}{1-c}$ then

$$\frac{\partial h}{\partial\varepsilon'_\gamma}\left(\tilde{E}'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\,\Omega_\gamma\right)=\frac{1}{\left[1+\dfrac{\varepsilon_\gamma}{1+\varepsilon_\gamma(c-1)}(1-c)\right]^2}=-[1+\varepsilon_\gamma(c-1)]^2 \tag{16}$$

Consequently, for given $\varepsilon_\gamma\in(0,\infty)$, $\Omega'_\gamma.\,\Omega_\gamma\in s^2$ and $r(\lambda)$, we get:

$$\int_o^\infty\tilde{\sigma}_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\,\Omega_\gamma\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)d\varepsilon'_\gamma$$

$$=\int_o^\infty\sigma_{c,\gamma}\left(\varepsilon'_\gamma,\Omega'_\gamma.\,\Omega_\gamma\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)\delta_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma\Omega'_\gamma.\Omega_\gamma\right)d\varepsilon'_\gamma$$

$$=\begin{cases}\sigma_{c,\gamma}\left(\in_\gamma,1\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,\varepsilon_\gamma\right); & if\ \Omega'_\gamma.\,\Omega_\gamma=1,(\Omega'_\gamma=\Omega_\gamma)\\ \dfrac{\sigma_{C,\gamma}\left(E'_\gamma,c\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,E'_\gamma\right)}{[1+\varepsilon_\gamma(c-1)]^2}; & if\ \Omega'_\gamma.\Omega_\gamma=\cos\theta=C\neq1, and\ \varepsilon_\gamma\in\left(0,\dfrac{1}{1-C}\right).\\ 0\quad; & \text{othewise}\quad,\left(C\neq1,\text{ and }\varepsilon_\gamma\geq\dfrac{1}{1-C}\right)\Leftrightarrow\left(C\leq1-\dfrac{1}{\varepsilon_\gamma}\right),\end{cases} \tag{17}$$

Where $\tilde{E}'_\gamma=\dfrac{\varepsilon_\gamma}{1+\varepsilon_\gamma(C-1)}$.

Note that equation (17) define for fixed Ω_γ a function on S^2 the unit sphere. When the energy ε_γ belongs to $\left(0,\dfrac{1}{2}\right]$ then $C\leq1-\dfrac{1}{\varepsilon_\gamma}$ is not possible and the support of the function (17) is the whole sphere S^2 but when the energy ε_γ is larger than $\dfrac{1}{2}$ then the support of (17) is only part of S^2, as shown in Figure 3.

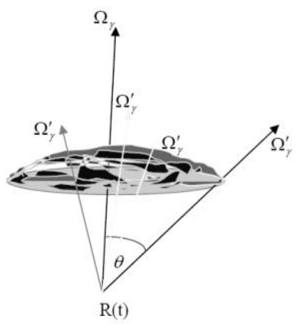

Fig. 3

Therefore we get from the above equation (17)

$$g^{(i-1)}_{\varepsilon_\gamma,\Omega_\gamma}(\lambda)=\rho_e(r(\lambda))\int_o^\infty \int_{s^2}\tilde{\sigma}_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)d\Omega'_\gamma d\varepsilon'_\gamma.$$

$$=\rho_e(r(\lambda))\int_{s^2}\int_o^\infty \tilde{\sigma}_{c,\gamma}\left(\varepsilon'_\gamma,\varepsilon_\gamma,\Omega'_\gamma.\Omega_\gamma\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)d\varepsilon'_\gamma d\Omega'_\gamma$$

$$=\rho_e(r(\lambda))\int_D \frac{\sigma_{c,\gamma}\left(E'_\gamma,C\right)\psi_\gamma^{(i-1)}\left(r(\lambda),\Omega'_\gamma,\varepsilon'_\gamma\right)}{\left[1+\varepsilon_\gamma(C-1)\right]^2}d\Omega'_\gamma. \tag{18}$$

Where $D\subset S^2$ is defined by $D=\left\{\Omega'_\gamma\in S^2:\Omega'_\gamma.\Omega_\gamma=\cos\theta=C\neq 1 \text{ and } \varepsilon_\gamma\in\left(0,\frac{1}{1-C}\right)\right\}.$

2.2 Change to spherical coordinates

We change the equation (18) to spherical coordinate in here. We take s^2 as a unit sphere then, for point $(x,y,z)\in s^2$, we can put, $x=\rho\sin\varphi\cos\vartheta$, $y=\rho\sin\varphi\sin\vartheta$ and $z=\rho\cos\varphi$ where $\rho=1$, φ is the zenith angle with $0\le\varphi\le\pi$ and ϑ is the polar angle with $0\le\vartheta\le 2\pi$.

Now,

$$T_\varphi=\left(\frac{\partial x}{\partial\varphi},\frac{\partial y}{\partial\varphi},\frac{\partial z}{\partial\varphi}\right)^T=(\cos\varphi\cos\vartheta,\cos\varphi\sin\vartheta,-\sin\varphi)^T \tag{19}$$

$$T_\vartheta=\left(\frac{\partial x}{\partial\vartheta},\frac{\partial y}{\partial\vartheta},\frac{\partial z}{\partial\vartheta}\right)^T=(-\sin\varphi\sin\vartheta,\sin\varphi\cos\vartheta,0)^T \tag{20}$$

$$T_\varphi\times T_\vartheta=\begin{bmatrix} e_1 & e_2 & e_3\\ \cos\varphi\cos\vartheta & \cos\varphi\sin\vartheta & -\sin\varphi\\ -\sin\varphi\sin\vartheta & \sin\varphi\cos\vartheta & 0\end{bmatrix}$$

$$=\left(\sin^2\varphi\cos\vartheta,-\sin^4\varphi\sin\vartheta,\cos\varphi\sin\vartheta\right) \tag{21}$$

Therefore,

$$\left\|T_\varphi\times T_\vartheta\right\|=\sin\varphi \tag{22}$$

The given conditions are $\Omega'_\gamma.\Omega_\gamma=\cos\theta=C\neq 1$ and $\varepsilon_\gamma\in\left(0,\frac{1}{1-C}\right)$.

Now we get from the above line

$$\varepsilon_\gamma\in\left(0,\frac{1}{1-C}\right)\Rightarrow 0<\varepsilon_\gamma<\frac{1}{1-C}$$

$$\Rightarrow\frac{1}{\varepsilon_\gamma}>1-C\Rightarrow C>1-\frac{1}{\varepsilon_\gamma} \tag{23}$$

Therefore,

$$1-\frac{1}{\varepsilon_\gamma}<\cos\theta<1,\ \text{since}\ \cos\theta\neq 1. \tag{24}$$

Since C is a function of $\Omega'_\gamma.\Omega_\gamma=\cos\theta=c\neq 1$ and $\in_\gamma\in\left(0,\frac{1}{1-c}\right)$. and E'_γ is a function of C so we write $C(\Omega'_\gamma)$ and $E'_\gamma(\Omega'_\gamma)$ instead of C and E'_γ accordingly in the following $H^{(i-1)}(\Omega'_\gamma)$.

Let $H^{(i-1)}(\Omega'_\gamma) = \dfrac{\sigma_{C},_\gamma \left(E'_\gamma(\Omega'_\gamma), C(\Omega'_\gamma)\right)\psi_\gamma^{(i-1)}\left(r(\lambda), \Omega'_\gamma, E'_\gamma(\Omega'_\gamma)\right)}{[1+\varepsilon_\gamma (C(\Omega'_\gamma)-1)\ ]^2}$ (25)

and $\mathrm{D}_1 = \mathrm{supp}\,\psi_\gamma^{(0)}$

Where supp stands for "Support".

The explanation of support: Let $f : S^2 \to \Re$ be a function. Then the support of this function say supp f is defined by supp supp $f = \overline{\{\Omega_\gamma \in S^2 : f(\Omega_\gamma) \neq 0\}}$.

From the energy condition we get

$$1-\frac{1}{\varepsilon_\gamma} < \cos\theta < 1. \tag{26}$$

If we look at (26) when $\varepsilon_\gamma \in \left(0, \frac{1}{2}\right]$ then the inequality is true for every θ, except θ=0. In this case, D will be the total S^2 and, for $\varepsilon_\gamma > \frac{1}{2}$, D will be $D \underset{\neq}{\subset} S^2$ (like an umbrella, see Figure 3).

Therefore, by considering the above condition we get from the equation (18) by changing to spherical coordinates when $i=1$,

$$g^{(0)}_{\varepsilon_\lambda,\Omega_\lambda}(\lambda) = \begin{cases} \rho_e(r(\lambda))\iint_{D_1^*} \hat{H}^{(o)}(\varphi,\vartheta)\sin\varphi\, d\varphi\, d\vartheta \text{ when } \varepsilon_\gamma \in \left(0,\frac{1}{2}\right], \\ \rho_e(r(\lambda))\iint_{(D_1\cap D)^*} \hat{H}^{(o)}(\varphi,\vartheta)\sin\varphi\, d\varphi\, d\vartheta \text{ when } \varepsilon_\gamma > \frac{1}{2}, \end{cases} \tag{27}$$

Where $E'_\gamma = \dfrac{\varepsilon_\gamma}{1+\varepsilon_\gamma(C-1)}$ and $\mathrm{C} = \Omega'_\gamma.\Omega_\gamma \neq 1.$

If we define the function $\zeta : (0,\pi)\times(0,2\pi) \to s^2$ given by $\zeta(\varphi,\vartheta) = (\sin\varphi\cos\vartheta, \sin\varphi\sin\vartheta, \cos\varphi)$ then $D_1^* = \zeta^{-1}(D_1), (D_1 \cap D)^* = \zeta^{-1}(D_1 \cap D)$ and $\hat{H}^{(0)}(\varphi,\vartheta) = H^{(0)}(\zeta(\varphi,\vartheta)).$

For easy calculation we have used for the integrating area the minimum rectangle containing D_1^* for $0 < \varepsilon_\gamma \leq \frac{1}{2}$ and the minimum rectangle containing $(D_1 \cap D)^*$ for $\varepsilon > \frac{1}{2}$.

For $i = 2,3,\ldots\ldots,M$ we get

$$g^{(i-1)}_{\varepsilon_\lambda,\Omega_\lambda}(\lambda) = \rho_e\left(r(\lambda)\right)\iint_{D^*} \hat{H}^{(i-1)}(\varphi,\vartheta)\sin\varphi\, d\varphi\, d\vartheta$$

for $\varepsilon_\gamma \in (0,\infty)$. Here also $D^* = \zeta^{-1}(D)$.

3 Conclusion

We have used the Compton scattering cross section in our Boltzmann model for solving the system of photon equation. The differential scattering cross section are differential in energy and in solid angle. The Compton scattering cross section can be decomposed into a product of a cross section, that is only differential in solid angle or energy and a Dirac delta function. Total cross section is calculated by integrating the double differential cross section with respect to energy and solid angle. Because the Delta functions one integral is always trivial.

To represent the cross-section we have used the quantities with a prime for incoming particles and the quantities without prime for outgoing particles. We have used the following symbols;

(a) for incoming energy we have used ε'_γ;

(b) for outgoing energy we have used ε_γ;

(c) for incoming direction of photon we have used Ω'_γ ;

(d) for outgoing direction of photon we have used Ω_γ ;

The mathematical model for the solution of an improper integral is described in this paper. This integration is needed for the solution of the Boltzmann transport equation. For reducing the calculating time, we have used the possible minimum rectangle for the integration area.

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