**JORDAN DERIVATIONS ON LIE IDEALS**

**OF -PRIME RINGS**

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**ABSTRACT**

In this paper we prove that, if *U* is a -square closed Lie ideal of a 2-torsion free -prime ring *R* and is an additive mapping satisfying for all then  holds for all

**Keywords:** Lie ideal, -square closed Lie ideal, -prime ring, Jordan derivation, derivation.

**1. Introduction**

Throughout the paper, we consider *R* to be an associative ring with centre *Z*. which denotes the commutator of *a* and *b*, we will use the identities: and for all . An additive subgroup *U* of *R* is called a Lie ideal if An additive mapping is called a derivation if holds for all and it is called a Jordan derivation if holds for all Clearly every derivation is a Jordan derivation but the converse is not true in general. A ring *R* is said to be a prime ring if  implies that *a =* 0 or *b=* 0*.*  An additive mapping is called a generalized derivation with the associated derivation if holds for all it is called a Jordan generalized derivation with the associated derivation *d* of *R* such that holds for all . R. Awtar[1] proved that if is a square closed Lie ideal of a 2-torsion free prime ring *R* and is an additive mapping such that for all then holds for all

We need the following lemmas due to R. Awtar[1] for proving our result.

**Lemma 1.1** If is a Lie ideal of a ring *R*, then holds for all

**Lemma 1.2** If is a Lie ideal of a ring *R*, then holds for all

**Lemma 1.3** If is a Lie ideal of a ring *R*, then holds for all

**Lemma 1.4** If is a Lie ideal of a ring *R*, then  holds for all , where

**Lemma 1.5** If is a Lie ideal of a ring *R*, then for all , where is as in Lemma1.4

**2. Jordan Derivations on Lie Ideals of -Prime Rings**

Let *R* be a ring. A mapping is called an involution if and holds for all A Lie ideal *U* of *R* is called a -Lie ideal if and it is called a -square closed Lie ideal if it is a -Lie ideal and for all A ring *R* with involution is said to be a -prime ring if implies that *a=*0 or *b=*0. It is worthwhile to note that every prime ring having an involution is -prime but the converse is not true in general. As an example, let where is an opposite ring of a prime ring *R* with involution Then T is not prime if But, *R* is -prime if we set and then and and thus by Oukhtite and Salhi [6]. We define the set which are known as the set of symmetric and skew symmetric elements of *R*. Let *U* be a Lie ideal of *R.* We define which we shall call the centralizer of *U* with respect to *R*. Oukhtite and Salhi [12] worked on left derivation on -prime rings and proved that or , where *U* is a nonzero -square closed Lie ideal of *R*. Oukhtite and Salhi [12] described additive mappings such that , where *U* is a nonzero -square closed Lie ideal of a 2-torsion free -prime ring *R* and prove that for all *.* Afterwords, Oukhtite, Salhi and Taoufiq[11] studied Jordan generalized derivations on -prime rings and proved that every Jordan generalized derivation on *U* of *R* is a generalized derivation on *U* of *R*, where *U* is a -square closed Lie ideal of a 2-torsion free -prime ring *R*. Some significant results on Lie ideals and generalized derivations in -prime rings have been obtained by M. S. Khan and M. A. Khan [5]. On the other hand, various remarkable characterizations of -prime rings on -square closed Lie ideals have been studied by many authors viz. M. R. Khan, D. Arora and M. A. Khan [4] ; Oukhtite and Salhi[7, 8, 9,10] and J. Bergun, I. N. Herstein and J. W. Kerr [2] and I. N. Herstein [3]. In this paper, we shall prove that if *d:R R* is an additive mapping satisfying , where *U* is a -square closed Lie ideal of a 2-torsion free -prime ring *R* then for all and hence every Jordan derivations on a -prime ring *R* is a derivation on *R*. We begin with the following results.

**Lemma 2.1**  Let *R* be a 2-torsion free -prime ring and *U* be a -Lie ideal of *R*. Let be any element such that for all then

**Proof*:*** We have for all . Let , then *.* Replacing *x* by *xy*, we have So

Since *R* is 2-torsion free so . For every we have . Putting *zx* for *y*, we have Therefore,

Therefore, Since we have *,* for all *.* Let Then If and then

Hence By the -primeness of *R,* we get

**Lemma 2.2** Let *R* be a 2-torsion free -prime ring and be a -Lie ideal and a - subring of *R*. Then either or *U* contains a nonzero -ideal of *R*.

***Proof:*** First we assume that, *U* as a -ring is not commutative. Then for some and Therefore the ideal *J* of *R* generated by is nonzero, and On the other hand, let us assume that *U* is commutative. Then for every for all Hence by Lemma 2.1, This shows that

**Lemma 2.3** If is a -Lie ideal of a -prime ring *R*, then

***Proof:*** is both a -subring and a -Lie ideal of *R* and contains no nonzero -ideal of *R*. In view of Lemma 2.2, Therefore,

**Lemma 2.4** If *U* is a -Lie ideal of a -prime ring *R* and *aR*. If then that is,

***Proof:*** If then by Lemma 2.3, so a centralizes *U.* On the other hand, let then we have for and. In view of Lemma 2.1*,*  This yields that For both the cases we have seen that This gives that

**Lemma 2.5** Let be a -square closed Lie ideal of a 2-torsion free -prime ring *R* and be an additive mapping satisfying *,* for all . If for all  then for all .

***Proof:*** In view of Lemmas 1.4 and 1.5, we have This yields that by Lemma 2.4. Hence for every , we have

**Lemma 2.6** ([7], Lemma 2.2) Let be a -Lie ideal of a 2-torsion free -prime ring *R* and such that then

**Theorem 2.7** Let *U* be a -square closed Lie ideal of a 2-torsion free -prime ring *R* and be an additive mapping satisfying for all , then holds for all *.*

***Proof:*** If *U* is a non-commutative Lie ideal of *R* then By Lemma 2.5, we have for all . Let us assume that Since we have as *.* If and , then Also, if and then Therefore, we have By applying the Lemma 2.6 in the above relation, we obtain that or for all Let and Then and are additive subgroups of U such that . Then by Brauer's trick or . Using the similar argument, we have  or  If then , which yields that , by Lemma 2.2. Which is a contradiction to the fact that So we have  and hence for all This implies

Now let If we define  Then and we have  Therefore, in view of (1), we obtain

Thus  Since *R* is 2-torsion free, we obtain

If *U* is a commutative -Lie ideal of *R*, then by Lemma 2.2, Therefore, by using 2-torsion freeness of *R* and in view of the Lemma 1.1, we have

In view of above theorem, we obtain the following corollary.

**Corollary 2.8** Let *R* be a 2-torsion free -prime ring. Then every Jordan derivations on *R* is a derivation on *R*.

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